[The 2012 Kellerman Lecture for New South Wales and Australia Capital Territory, delivered at the eleventh conference of the Australian and New Zealand Masonic Research Council, at Wellington, New Zealand, on 15 November 2012, and published in *ANZMRC Proceedings 2012*. Since then the author has continued to expand and update this paper, with the intention of publishing it as a book.]

# STATISTICAL MECHANICS OF THE MASONIC MIND

by Philip Purcell

#### **Abstract**

Statistical probability (mechanics) theory, borrowing from the concepts of thermodynamics, defines an algebraic theorem of the evolving Masonic mind. A topology of a network of societies is mapped for the investment in the algebraic symbolism of meta-knowledge (the self awareness of knowledge) and the purpose of ritual. Traced is the Masonic mind in transition, from the limiting degrees of freedom of a culture normalising *a posterior* expectations, toward the disinterested exploration of uncertain interior *a priori* probabilities.

Statistical modelling of the social capital of a network of 14 societies over 7000 years reveals choices between systems that look rearward (closed) or to the present (transitional) or to the future (open). Probabilities of expectations being satisfied are defined as space enclosing and packing fractal structures that are self-replicating and map as tri-level spirals. The dimension of the enclosing power law structure seeking to normalise events, is less than the interior open-ended logarithmic microstructure of the disinterested mind's multiple probabilities. Their bridging depends upon a sympathetic (homeomorphic) entropic or chaotic trigger.

Economic geography and demographic transition theories explain how socio-economic masses interact to capitalise on interactions between income, population, education, spatial separation, technologies, political-economy and external shocks. Their probabilistic statistics inform an algebraic topology of the network's meta-knowledge content. Dimensions map as a fractal structure then as a spiral graduation from rote learning of the simplex outer macrostructure to the complex inner microstructure of the disinterested Masonic mind.

## Questions

Research began with speculation as to whether the Masonic legend is amenable to quantitative research. This led to the testing of the application of statistical (mechanics) probabilities and sets to the dynamics of complex social systems. Borrowing from Czeslaw Marchaj on the *Aero-Hydrodynamics of Sailing*, 1979 (page 346), the proposition is: without some guiding idea or theory one cannot even determine which features or factors to look for. Is Masonic ritual entrapped by the axiomatic rather than exploring the symbolism of algebra as a gateway to a broadening consciousness? What of the modern re-drawing of brain connectivity for the software of information technology, so repositioning learning and ritual?

Mathematician George Boole (1815-1864) on *The Right Use of Leisure* (1847) then economist John Keynes (1883-1946) on *The Economic Possibilities for our Grandchildren* (1930) forecast a diminished utility from paid hours as workplace productivity rises, freeing time for higher learning (the ideal of antiquity). Demographers point to a window for today's complex societies for such a demographic dividend, as the three main stages of life balance out around the stabilising of higher incomes, life expectancies and educational opportunities. However, does society have the will to reappraise its choices to unlock the knowledge of a cerebral freedom? What inconvenient semiotic is revealed by contrasting nomenclatures like: complex society v. civilisation, grey ages v. dark ages, or interest groups v. harmonising?

Statistical modelling of the enduring factors or topology within political-economy, suggests that self-interested trading cultures normalise or hard-wire surface values of income and mobility, at the expense of the disinterested study of the mind's deeper multiple dimensions. Demographic and creative interactive media change agents, steer the network's software and connectivity, to broadcast non-linear dimensions, before being normalised.

Australian Bureau of Statistics *demographic* and *social trends* show that from 1920 to 2000 the gross wealth of Australians increased 6.5 fold and working hours fell then rose after 1980. Australians graduating from higher education as a proportion of the population increased from 3% to 16% over the period 1970 to

2000. On reaching the workforce more would be own-account workers rather than salaried. What then are the implications for the theory and practice of the reflective sanctuary of the Masonic lodging? Without a theoretical reworking, as unwelcome as that might be to invested tradition, can Freemasonry meet its challenges?

Euclidian and Newtonian analogies are implied in the Masonic symbolism of the enduring intrinsic properties of society. However, has this topology been sufficiently theorised beyond the practical analogy? Instead my thesis proposes a Masonic symbolism where the axiomatic is a simplex building block of a complex interior algebraic microstructure in a space packing fractal topology. There rising network complexity de-correlates the past and futures.

That such an algebraic combinational topology has been a bridge too far for Freemasonry has history, going back to the formative years of the early 1700's analysed in *The Key to Modern Freemasonry* (2011) by Charles Lawrence. Subsequently Leonhard Euler's (1707-1783) formulation of complex numbers in 1748, the 1874 set theory of Georg Cantor (1845-1918) and George Boole's algebraic logic, are beyond Masonic ritual. Missed opportunities include the lectures to university undergraduates and Masons by Cassius Keyser (1862-1947) on *Mathematical Philosophy* (1922), reviewed by *The* (Masonic) *Builder*. Later Morris Kline (1908-1992) wrote on the history of mathematical thought and education (*Mathematics in Western Culture*, 1954), yet today academics point to a limited mathematical literacy in the general community. Even the appreciation by mathematicians of theoretical advances is necessarily lagged. Set theory of Pierre Fatou (1878-1929) and Gaston Julia (1893-1978) preceded the fractals of Benoit Mandelbrot (1924-2010). Henri Poincare's (1854-1912) work preceded the *Essence of Chaos* (1993) of Edward Lorenz (1917-2008). Complex interactions are also confounding. Topology, manifold, set, and vector theories connect with the probabilistic statistical mechanics of Josiah Gibbs (1839-1903) and Richard Tolman (1881-1948). In turn they integrate concepts from the thermodynamics of Ludwig Boltzmann (1844-1906), and the non-linear mathematics of Rudolf Clausius (1822-1888) and Bernhard Riemann (1826-1866).

To what extent does society and the Masonic sub-culture appreciate that: the fundamental nexus is between the self-interested expectation of a harmonious orderly grand design, that contrasts with an orderly understanding of the dynamic harmonics of meta-theory and the uncertainty in the non-linearity of system behaviour? Entropy primes the creative chaos of innovation, as with factor constraints of the plagues and mini ice ages, the former transmitted by the Silk Road. How does Freemasonry reconcile its potential as a conduit of meta-theory with its practice as a genteel social club, where geometric symbolism is crafted into ritual to convey moral or ethical standards? Has Freemasonry sufficiently embraced the notion of the areté of Socrates (469-399 BCE), Plato (429-347 BCE), Aristotle (384-322 BCE) then Alexander the Great (356-323 BCE)? Their public good was the classical route of study into higher learning, from which virtuous living, expressed by the likes of charitable works or religious observances, follows.

## **Concepts**

**Framework**: Demographic modelling describes the economic history of a network of 14 societies over 7000 years for the choices made by societies where their cultures are defined by set theory. System stability is contrasted with the transformative dynamic of entropy or chaos. This algebra of statistical mechanics, drawing upon physics and the social sciences, defines the network's coefficients (constants) and metrics (variables). These map as a topology of the social capital (the product of social interaction) that is located within the cultural settings. Interacting network connectivity and software are then displayed as fractal intervals within a geodesic spherical analogy of the potential of the Masonic mind.

Demographic gravitation theory pioneered in 1948 by astrophysicist John Stewart (1894-1972), applied in new trade theory in economic geography, describes socio-economic masses of trading societies organising to overcome spatial and cultural separation. Demographic transition theory pioneered by Warren Thompson (1887-1973) in 1929 describes the socio-economics of transitions in demographic profiles for their political-economy, education and income distributions. Framed by the statistical probabilities, sub models also assess the economic history of the network's connectivity, system shocks (e.g. war, famine, climate change) and technology. These render a statistical view of the symbolic Masonic outer and inner chambers, Boolean gateways, mosaic carpet, spiral stairs, entropy and rebirth.

Similarities, relationships and different responses among societies are assessed. For example, Mesopotamia's value adding was a mix of climate, river systems and the skill of landowners but not the slave labour of Athenians. Mesopotamian warring city-states were a product of isolation and the need for

farming land whereas Greek states were a defensive coalition against Persia, before turning inward. Hellenic culture owed much to the Ionians who escaped the Dorian conflict 1000 BCE to establish city-states in Anatolia. 1300 years later these were drawn into the orbit of Constantinople founded in 330 CE as the epicentre of the Roman Byzantine Empire with its Imperial library, in the tradition of Alexandria, until destroyed in 1204 CE by Crusaders. Another conservatory was Baghdad's House of Wisdom c.800 CE industrialising papermaking, innovating universities and al-Khwarizmi's algorithmic solution to general problems. Mongols sacked Baghdad in 1258 CE but effected a cultural restoration, although intellectual tolerance qualified by State policy based on Islamic values, would not withstand the industrial-scientific power of the Western mercantile model.

A theorem of Masonic ritual and its education encapsulates the content and awareness of the enduring social topology of the statistical analysis, where ritual is the shell of an algorithm. The cultural contest is between the exploitative expecting to *normalise* income and mobility, or a disinterested exploration of the *uncertainty* within meta-theory. Modelling identifies the change agents as demographics and the media broadcasting the possibilities of a non-linear reality. Non-linearity in dynamic systems, proved by Henri Poincare's solution to the question of the stability of the solar system (the Three Body Problem), is the statistical possibility that changes in initial boundary conditions, have unpredictable consequences.

**Set theory:** Network hubs and nodes are statistically modelled for their political-economy by analogy with the geometric cross section of cones to distil their social freedom and the choices made. Such sets may be closed (circle), sub sets (elliptical), partial unions (parabolic), or replicating unions (hyperbolic). A break down of the boundary conditions of the topology opens up a non-linear conjunction of entropic and chaotic precursors of multiple futures.

Quadratic discriminants (the eccentricity of conical cross sections) of the cultural sets map as complex numbers (real and imaginary) of Leonhard Euler's analytical mathematics, for the mirror imaging of number l capital, depending upon the openness of societies. This is tested for the coefficients of variation about the line of best fit, and the probabilities of expectations being realised. Such are affected by each society's propensity for equality in the distribution of income, demographics, educational opportunity and thus their liberating possibilities.

Set theory aligns the linear data sets by attributes for testing: the probability of the effect of individual factors on expectations (material v. meta-theory) being realised (e.g. mean values, standard deviations) and null hypothesis testing of the degrees of freedom under the normal curve; the probability of multiple factors agreeing or not (e.g. regression and co-relationship analysis); and the transitions by Boolean (truth table logic) gate arrays for data sets. Cultural replication is presented as an adapted Benoit Mandelbrot quadratic fractal based on complex numbers, producing fractal dimensions locating ritual within the system dynamics.

Socio-economic data: Multivariate analysis encounters the issue identified by Ian Morris and Joe Manning on the interdisciplinary study of the economics of Mediterranean antiquity (*The Ancient Economy: Evidence & Models*, 2007). Ancient historians and archaeologists amass material that is difficult to theorise because the data is thin and specialties are compartmentalised. That compounds a divide between the historian's interests in the rich tapestry and the social scientist interrogating the underlying fabric. Due to the scarcity of hard data for the societies of antiquity my approach is to identify data markers and indicators. Derived from trade cycle theory, archaeological material such as numismatics and descriptive cultural histories are scaled, interpolated and tested by probability analysis.

**The product**: A statistical picture is built up of the principal factors driving the choices and achievement of the network. Driven by material needs and wants the contrast is with an alternative society driven by the want for meta-theory, and what that means for the theorem and practice of Masonic lodging. Social capital is mapped as a fractal formation and compared with the scale of fractal dimensions. Emergent is a view of the network's social connectivity relative to the neurological potential mapped as an analogue of Jerome Bruner's spiral learning curve. There, the social capital of society is transcended by the potential of the Masonic mind, influenced by Carl Jung's (1875-1961) collective self-knowing culture.

Within the network complexity, patterns de-correlate and the propensity of society is to regress to a perfected ritualised outer shell, blind to an alien freedom in an understanding of the uncertain multiple realities within. A simile is homomorphic encryption, pioneered by Serge Vaudenay *Decorrelation: A Theory for Block Cipher Security* (2003) and developed at IBM by Craig Gentry as *A Fully Homomorphic Encryption System* (2009). Breaking from the mutually exclusive degrees of freedom under the normal curve, inputs and outputs do not modify the algorithm. Being independent of random sampling the toss of a

coin does not change the probabilities of decryption. This Andrei Markov (1856-1922) chain means that the past is decoupled from the future with its potential for social transformation.

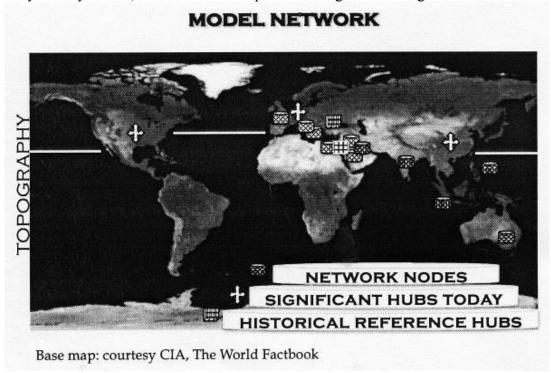
Laws of Form of George Spencer Brown of 1969 disassociates content and form to redefine relationships among knowledge domains, furthering the insights of Kurt Gödel (1906-1978) and Bertrand Russell (1872-1970). Sets are unlikely to be unique or complete. Those that appear identical may contain particle differences. Similarly, new trade theory is not a zero sum game. Relationships build on product differentiation. Although negotiated relationships may be coercive of third parties, they produce new harmonies. Externalities are explained by the conditional logic of Charles Peirce (1839-1914). Switching intervals at Boolean logic gates mediate the spiral learning curve for the structuralist's cultural environment.

**Documentation:** This research paper is a synthesis of a working treatise detailing the historical information, its sourcing and the derivation of the statistical methods for its interrogation. Statistical relationships/coefficients, mapping and transformative operations are also documented with factor probabilities and sensitivities. Factors are tested individually and in combination to identify normalising or out riding tendencies. Design work is handraulic as the prototype model is developed. Also each data cell has a history.

**Bibliography:** Analysis of the full complement of reference material reveals strong domain vectored relationships between Melvil Dewey (1851-1931) classifications of science-mathematics v education-knowledge-information v the social sciences. Philosophy and history-geography present as enabling knowledge domains. Literature and the arts appear to be of minor importance but in a multi-media presentation this changes significantly. The rise of modern knowledge, information and their communication systems theory, is suggested by the time line of sources where the distribution is a normal curve from 1750 CE to the present time preceded by a slim tail out to 5000 BCE. The geographic origin of referenced material is: 45% USA; 20% UK; 15% Other European; 15% ANZ; and 5% rest of the world.

## **Modelling**

**Phase 1 (of 6) the proposition**: is that an historical demographic-media conjunction positions the network of societies to transition, from a normalising exploitative culture with limited degrees of freedom, to a liberated consciousness. Philosophical and quantitative components of social capital are modelled by statistical mechanics (set and probability theories) for their inertial linear and dynamic non-linear patterns. This is not to validate a posterior interests but to liberate a priori possibilities for their patterns and symbolism, mindful of Keynes' Treatise on Probability (page 4) that a proposition is not probable because we think it so. Modelling seeks to reveal the enduring interior statistical structure of society and nature that is found in the uncertainty of system dynamics, and how this is expressed as a graduated degree structure.



**Phase 2** (of 6) the topography of the social-mass: is the social product of a network of 14 trading societies over 7000 years as statistically estimated. Probabilities of expectations being realised whether normalising or transforming are modelled. Nominal measures of national product are qualified by estimates of their real content having regard for income and demographic distributions intertwined with political organisation and cultural norms.

This *enumerator* of the demographic gravitational attractor of a network of societies is posited by modern economic geography and new trade theory, for the extraction of their intrinsic social capital. Societies modelled are: U.S.A.; Western Europe and separately UK, Greece and Italy; Egypt; Anatolia / Turkey; Mesopotamia / Iraq; Saudi Arabia (and the historical antecedence); India; China; Japan; Indonesia; and Australia.

This is essentially an extended Silk Road network but excluding the ancient route into the eastern or Sub-Saharan Africa, (these recently emergent economies being of particular interest to China and India).

**Nominal network product**: is the product of income per head and population. Post 01 CE metrics are sourced from the tables of Angus Maddison (1926-2010). The analytical task is to ensure that spot valuations are representative of historical trends and to derive metrics 01 CE to 5000 BCE. Analyses of Bradford DeLong (*Estimating World GDP*, 2000) and Michael Kremer (*Population Growth and Technological Change*, 1993) and the general historical record identify significant BCE break points. These can be interpreted for their technological, demographic, political, resource (e.g. wood supplies for ship building) and environmental or philosophical import. In my model, trend line estimations of pre 01 CE incomes and populations, are anchored by the data cells for that base year, spliced rearwards in 500-year intervals. Weighted by the slope of the global factor distributions, coefficients of the cycling of societies are superimposed. Chi Squared Chi2 testing refines the probable data chain for its probable best fit and sensitivity (degrees of freedom) to socio-economic factors and a tendency to normalise random events.

**Effective network product**: qualifies the raw social product, by the distributions of income and demographics, for the capacity of societies to invest in philosophy and open education.

Income distribution is a by-product of the political-economy. For example, Alexander the Great colonising NW India suppressed warring fiefdoms. His troops integrated with the local community which policy was also initially implemented by British colonisers until the mercantile core came through also disestablishing the administrative legacy of India. The Enlightenment had a mercantile core. In the age of John Locke (1632-1704) public education was bad policy. Adam Smith's (1723-1790) *The Wealth of Nations* 1776, the first systematic economic model, exorcised the excesses of mercantilism. That substituted one class structure for another, either being an exploitative imperialist culture. Equality and the inclusiveness of the political-market-economy may be a proxy for the freedom to reach for our potential. That is qualified by the Faustian propensity to delegate Fyodor Dostoyevsky's (1821-1881) dreadful responsibility of freedom to interest groups. *The Great Deceit of the Theory of an Enlightened Freemasonry* (2009) is Fabio Venzi's critique of a Masonic lineage to empiricism.

Coefficients representing social structures are averaged for their political-economy attributes, variously mixing: egalitarian; grey age (including imperial, command, colonising); laissez faire and mercantile; classical synthesis; moderne; and the post moderne multiple realities. Reference points are the complex forms of egalitarian and post moderne. Examples of countervailing non-linear factors interacting with political-economy and demography are:

Climate and natural catastrophes: beginning with warm ages 10,000 years ago facilitated complex cultures from around 5000 BCE in the middle northern hemisphere latitudes. Gradual cooling led into a significant temperature drop around 2500 BCE before rising in the Holocene inter-glacial period then with mini-ice ages returning 1650 CE to 1850 CE.

Famine: can be a consequence of drought, natural disasters, the breakdown of governance, distribution system bottlenecks (the ancient port of Rome) and the demands of urbanisation. Arne Wossink found that Climate Change Does Not Always Lead to Conflict (2009). During ancient mini-ice ages the exhaustion of land carrying capacity and the potential for war was mitigated through the trading systems such as with the Amorites of Mesopotamia.

*Plague / pandemic*: is a product of unsanitary and crowded conditions whether the intensive farming of animals or urbanisation, transmitted through trading networks. A response to The Black Death of the Middle

Ages CE, that decimated the worker and managerial classes, was productivity improvements, also necessitated by a concurrent mini ice age and famine.

*Warfare*: on *The Culture of War* (1990) Richard Gabriel found that even with the onset of arid conditions 4000 BCE, it was after the mini ice age 2500 BCE that warfare became more extensive.

Economic surpluses of the Fertile Crescent of the Mesopotamian Bronze Age brought a legacy of technological change, armies, development of cavalry and maritime power. The Iron Age brought structural change in the composition and tactical use of military force. The longest running conflict was between Rome and Persia 90 BCE to 630 CE.

Security: provided by extensive empires like the Greats, Cyrus (600-530 BCE) then Alexander (356-323 BCE), was made possible by professional administration, tolerance and cultural mixing. Greeks the professional soldier had force multiplying organisation, skills, technology and tactics. Later as Rome receded so did security of the Silk Road, reviving in the Middle Ages CE by the extensive Mongol Empire that was a conduit for European exploration.

Demographic transition theory traces the demographic profile of societies through four stages (I to IV), from high death/high birth rates to low death/low birth rates. During stage I children receive no formal education, are working and not dependent. By stage IV dependency ratios are high, reflecting extensive efforts to educate the young while longer living persons opting out of the workforce seek re-education. Dependency ratios do not necessarily distinguish between voluntary and involuntary dependencies.

The demographic dividend of stage III is a 30 to 40 year window where the dependency ratio of the non-working population is low prior to the assertion of longevity on the demographic profile. Parenting is no longer driven by child mortality rates or a family infrastructure.

Thus primary coefficients are the distribution of life expectancies and age profiles in populations, weighted by the average of their political-economy attributes. Stage I is a system at sub-optimal equilibrium. Stage II is a system in transition toward an optimal equilibrium. Stage III is a system at the United Nations optimal equilibrium. Stage IV is the inevitable entropic transition, evidenced today in Japanese and Western European societies.

Coefficients of dynamic lift are applied to the effective national product of societies to represent the compounding effect of the circulation of factors of production, investment and money supply as geared by the openness of political-economy and factor productivity. Dynamic lift is conceptually the core national product's tangential velocity, borrowing from the fluid mechanics of Daniel Bernoulli (1700-1782) and Heinrich Magnus (1802-1870).

**Phase 3** (of 6) the spatial and media topography: is the denominator in the modelling of the network's demographic gravitational attractor of societies. The system's physical transport infrastructure and embedded logistics and technologies array as a hub and spokes model. Spatial relationships of major multimodal nodes are defined in relation to the historical transit hubs of Port Said and Constantinople and the performance of air, land and sea platforms. Their metrics are merged with the technological history of media broadcasting the content, meaning and reach of the software of that transmission agency of cultural capital, where the graduations of George Boole's universal logic are transition points.

**Transport system** velocity made good is the vectored spatial hard-wired relationship between nodes relative to reference points. Mobility is defined by distance, technology and the embedded infrastructure and logistic organisation. Differential rates of factor change are arbitrated by the tangential sine wave of a propagating or limiting wave of Georg Ohm (1789-1854). This linear measure of the hub and node differentials of networked factors is geodesic as in the non-Euclidian great circle of Bernhard Riemann geometry. Even with the minimal path of the Ronald Fisher (1890-1962) *z* transformation, the geometry is superficial.

Illustrative of technology, cargo ships of Alexander the Great approaching 500 tonnes displacement are compared with a modern handy size ship at 32,000 tonnes dead weight and the modern ultra large crude oil carrier of 550,000 tonnes dead weight. The ultimate sailing ship the Bermuda rigged Clipper had the passenger capacity of a Boeing 747 that is 35 times faster, takes a more direct route, has a larger fleet and thus superior turnaround-back loading. Equalisation of rail and sea cargo capacities occurred in the 1940s, with the Chesapeake & Ohio Allegheny locomotive servicing the Liberty ships through multi-mode hubs of the USA. Each generation of transport systems lowers real unit transportation costs.

**Broadcasting media** is a software entity bootstrapping to infrastructure hardware, alignment and broadcasting signal strength of nodes and network. Elizabeth Eisenstein argues that *The Printing Press as an Agent of Change* (1979) related to broad industrial and cultural revolutions and is a revolution with a unique history of the understanding of events as with any media or communications technology. Communication systems are a metaphor of coding and symbolise ideas captured on progressively more durable and communicative media. This liberates human autonomous thought, reflective capabilities, social networking, builds social capital and multiplies the intensity of the symbolic content and meaning of the network.

Value added by broadcasting media is imposed on a rising base line of the physical broadcasting capability of the network. The percentile propensity to be a transceiver of intangible cultural cargo, that adds unique value, is defined by the representation of the discriminant of cultural sets by conical cross section, in relation to the openness of societies.

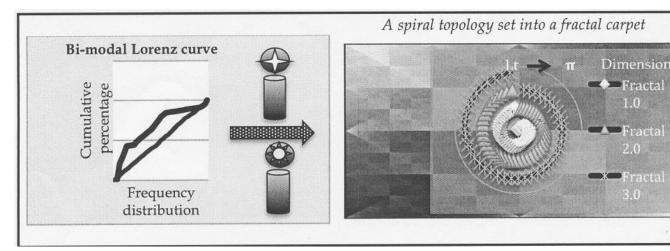
Base line physical broadcasting capability and the network's technological coefficients are a function of technology (scribing, early printing, electro mechanical and then digital), inputs (encoding, value added services, infrastructure and replication), and outputs (speed, reach, capacity and durability). Estimation of he coefficients and metrics are a function of the factor regression on an inverted quadratic discriminant (the eccentricity of conical cross sections). At the core of the second-degree formulation are imaginary numbers rotating the topology for the social capital of the multiple realities. Regression analysis provides the coefficient of percentile variation, by the method of least squares, about the line of best fit for the nodal data series, in relation to the network. Comparing the discriminant with the regression coefficient produces a data series for the network's virtual cultural transmission. Arrayed by spliced blocks of time the curve is smoothed around natural break points in the time series.

To test the reliability of the probabilities of expectations being satisfied, the minimal path of the variance in the co-relationships of independent data streams is estimated, by the Fisher *z* inverse transformation, yielding an indexed series of complex numbers. As with the Markov chain, the multiple future realities separate from the past. Matched with Boolean logic states, NOT (multiple realities) or NOR/NAND (axiomatic) replicators map to Felix Hausdorff (1868-1942) fractal dimensions. NOT closed sets decaying in a non-linear entropy fold over the open Edward Lorenz (1917-2008) strange attractor (chaotic) topology. Intermediate states OR support the containment of sets while AND is a reinforcing attractor.

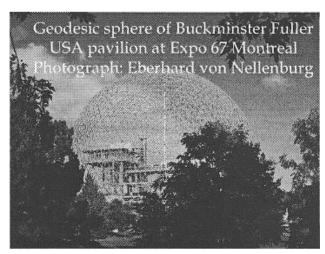
**Phase 4** (of 6) the fractal topology: maps the reach of societies as the vector quantity of the socio-economic mass and the network's spatial and media connectivity. This doubled between 5000 and 500 BCE at which point the network's 7000 year average was achieved. The quantity doubled again by 1500 CE and then six fold again during the next 500 years. Pre-eminent societies were: 500 BCE China, India and Western Europe; 1500 CE China; 1600 to 2000 CE the USA and Western Europe. Complex cultures of antiquity, bordering the Mediterranean, The Gulf and Red Sea are understated by their socio-economic mass.

Negative correlations within the pairings of income v. demography then transport (mobility) v. communications, suggests that commerce and leisure transport crowd out meta-theory. Also the statistical testing shows that the popular expectation that externalities can be normalised contrasts with the statistical reality of a rising deviation from mean values and a weakening of the regression line of best fit. Hence, income and transport pair to normalise expectations, while the dynamics of demographics and cultural communication stand aside the axiomatic network as carriers of meta-theory. The statistical solution is the mapping of the data for the cultural reach of societies by a matrix of 210 data points (14 societies x 15 time blocks) resorted for the cumulative share of the network's product. This presents as a reciprocal bi-modal variant of Max Lorenz's (1876-1959) curve of the inequality of wealth distribution. Here is the curve shifting of expectations, from a tri-tiered fractal power law topology of the exterior social capital, to the interior logarithmic cognitive spiral.

A coefficient of the rotation or orientation of the bi-modal curve may be derived by inspection, yielding a ratio of the y-axis rise over the x-axis of the data curve, on a base line that can be likened to the low level cosmic static that speaks of our distant cosmic origin. The rise is the difference between the initially steep divergent statistical plot, then normalised by rotations elsewhere along the curve, in relation to the saddle paralleling the reference line at 45 degrees. The later rise is a curve shifting. This visual assessment is an Occam's razor for the complex derivation of the number cis  $(\theta) = \cos\theta + i \sin\theta$  formulated by Leonhard Euler where the area under the curve is converted to a circular moment and area by trigonometry.



This maps as a Waclaw Sierpinski (1882-1969) mosaic carpet transcribed by the scale of topologist Felix Hausdorff (1869-1942) that demarks the transition to a complex interior microstructure. The perimeter of the exterior carpet breaks down, illustrated by the modernist chequer board stepping stones of the Mirei Shigemori moss garden (1939) of Kyoto (Tofuku-ji). This structural limit of the network's geodesic sphere hints at the graduated domain beyond the threshold of Masonic ritual. Logic gates of George Boole mimic the transition analogous to the Masonic pillars. Compliant with set theory the plane folds back onto itself to form a manifold or geodesic plane as the internal cognitive structure intensifies. Infinite intervals mean the journey advances but can never be completed.



Geodesic sphere of Buckminster Fuller

Formulation of the fractal architecture and its density is by a modified Mandelbrot self-similar quadratic polynomial equation whose base is complex numbers. Data sets increase by a margin for each box in a Sierpinski 5x5 matrix cycling about a core box on a base elevation. The core box comprises 162=256 data points that are thereby replicated by the abutment of boxes that fold over the core so shaping the density of the fractal within a fixed perimeter of rising pixel complexity. Hausdorff fractal dimensions H-D are derived by interrogating the pattern of the power series on a natural logarithmic base  $\log_e^{\# \text{iterations}} / \log_e^{\text{original }\# \text{boxes}}$ . Network historical choices and achievements produce a H-D of 1.5 with a self-imposed structural limit of 2.0 coinciding with the surface threshold of the fractal carpet and ritual. This compares with an absolute theoretical H-D limit of 3.0.

As a service organisation overlaying a spiritual core Masonry is defined by NOR/NAND switching points in Boolean logic describing the axiomatic in tension with the very idea of meta-theory. Above the dimension of 2.0 is a higher duality between entropy and rebirthing, their conjunction about the NOT Boolean switching threshold as their states wrap around. Entropy resets the network for rebirth by the survival imperative of innovation. Fractal intervals are of a power law kind. A fractal count of 2.0 is 9 times denser than 1.5. A dimension of 3 has a network connectivity 27 times denser than 1.5. Fractal perimeter and outline are defined by the coefficients of their simplex self-replication as they write over themselves and ever shortening internal connectors pack out the surface. The substrate is an interlocking Euclidian hourglass mosaic formation. Tri-level spiral piles of the Sierpinski carpet provide the Edward Lorenz strange attractor points for a transformation should a disturbance of the system's equilibrium render a non-linear (entropic-chaotic) sequence. The Lorenz *Deterministic Nonperiod Flow* (1962) shifts the H-D to between 2.0 and 3.0.

**Phase 5** (of 6) the consciousness: is conceived as the network's inner spiral curve as a function of the brain's fractal connectivity and the logarithmic software of its wave formations. James McClelland's *Memory as a Constructive Process* (2011) locates knowledge in the connections where memory attaches to symbols whether words, signals or relationships. Extrapolating from Paul Smolensky (website, 2012), is the Masonic rule-enclosing symbolism congruent with distributed processing and memory of networks? I am mindful of The Royal Society's caution on the harnessing of neuroscience in the service of learning systems.

Brainwave frequencies presenting as a Leonardo Fibonacci (c1170-1250 CE), logarithmic, accumulating spiral curve is the cultural software parent of the lesser power law fractals of blended hard and soft wares. Comprising the socio-economic modelling, they take the form of the Archimedean (c287-212 BCE) or Theodorus (465-398 BCE) square root spiral. Even so, the limits of their windings converge on  $\pi$ , being defined by the angle between the radius vector of the spiral and the reciprocal tangent, thus the distance or oscillation between cycles. Also allowing for the probabilistic nature of the modelling it is reasonable to assume that differences at the limits are not statistically significant, but it is the transition to the liberating consciousness that is steeper than suggested by the socio-economic phases of the modelling.

Spectral analysis drawing upon audio frequency filtration theory (high, low and band-pass) provides an analogue of the brain wave formations comprising frequencies and amplitudes, which may be additive, subtractive, vectored and phased. It is the ability to harness the non-linear formations that create new harmonies from the palate of frequencies, by actualising the chaotic strange attractor within the collective mind of the network. The mathematical microstructure is the complex function of impedance comprising: a *real* constant resistance of a conductor to energy flow that is invariant to frequency; and an *imaginary* reactance component difference between inductive and capacitance reactance (the charge stored) that is frequency sensitive. Their synchrony is a phase-shifting additive vector that in the modelling is framed as Felix Hausdorff discrete dimensional coordinates (1918).

Capacity to process and store information carried by these signals depends upon the hardware of the bundled neural fibre network connectivity of the central and autonomic nervous systems. Neurotransmitters and receptors communicate at junctional synapse by chemical-binding agents that are the plasticity of learning. Repetition up to a point strengthens associations. Switching points of the fast learning functionality, in conjunction with distributed fine tuned associative switching points of the learning brain, partition to protect the integrity of the structure of associative knowledge, as new relationships or information are tested and selectively absorbed. Connections encode and recall knowledge of the subjective experience in memory. Repeating recall declines dependency upon short-term memory in favour of connectivity patterns that rationalise. Their higher-level functionality integrates thinking, planning and memories. Current activity is either associated and filed to long-term memory or discarded after completion of current tasking.

Development of the brain's connectivity, as with the modelling of the network's economic geography, not only describes how brain wave oscillations as a soft media formulator are routed, but also why parts of the network are stimulated or shut down. The propensity for multiplexing system conflict and saturation may result in security seeking behaviour, so explaining the self-imposed limits of the network's potential. Joseph LeDoux' *The Emotional Brain* (1998) and Gregg Jacobs' *Ancestral Mind* (2003) interprets this as the overwhelming sociology of advancing cultures locking the neo cortex into a cycling crisis mode. Abraham Maslow's (1908-1970) *A Theory of Human Motivation* (1943) frames the contest as being between society's material deficiency (exterior) wants and higher being (inner) needs. Their complex number  $\gamma$  synonyms of  $\beta$  real and  $\alpha$  imaginary are the obverse of common usage.

Ascending brain wave frequency patterns are delta, theta,  $\alpha$  alpha,  $\beta$  beta and the composite  $\gamma$  gamma. Median wave patterns are within the Heinrich Hertz (1857-1894) range 5 to 25 Hertz and a composite limit of 40 Hertz. Hans Berger (1873-1941) Alpha (inner focused) frequencies oppose yet interacts with Beta (exterior awakened, focused, judgmental) to mark the fractal dimension 2.0, synonymous with the pillars at the threshold of Masonic ritual. Their symmetrical deviations from median frequencies are less than for other waves. Alpha in transition is an entropic precursor to the chaotic attractor of the Masonic third degree and the creative possibilities. Human evolution presently limits the fractal folding of the outer cerebral cortex to a dimension of 2.8 although the achievement is more like half that value.

Returning to the electronic analogy, polar electromagnetic patterns when hinged resemble the Edward Lorenz strange attractor or butterfly effect in chaos theory. That is mimicked by the probabilities of expectations being realistically realised for the modelled system factors like income, population, transport, communications and systemic shocks. Risk tolerance tested by null hypothesis contests the idea that normalising of random events is a sustainable substitute for the curve shifting that occurs with system entropy that primes innovation.

**Phase 6 (of 6) the theorem:** describes Masonic ritual as an algebraic algorithm that is a conduit for the mastery of multiple dimensions through our choices and cognitive wiring. A complex topology of the interior mind maps as a spiral of fractal intervals. Bridging with the exterior mosaic fractal formation of

Euclidian hourglass simplexes, mapped as a lesser spiral, depends upon the homeomorphic (continuum) trigger of a Mandelbrot strange attractor.

Analysis chisels away at the historical semiotic coatings to reveal the disinterested algebra of the Masonic mind and re-evaluates notions of *freedom*, *harmony* and *stability*. Society's homologating Euclidian and Newtonian axioms, ceding security in a group setting as a harmonious *orderly arrangement*, contrasts with the transformative harmonics of the potential alien inner freedom. Destiny is found in an *orderly understanding* of the algebra of the non-linear dimensions, produced by the network's de-correlating yet intensified connectivity.

Emergent is a rising self-awareness, beyond the self-interested deficiency needs described by Abraham Maslow. A trivial culture of food, fashion, or elaborate housing and gladiatorial stadium entertainment carries a high opportunity cost, that is secondary to their misappropriated symbolism of civilised living. Their obverse is a mislabelling of dark ages, because data is thin or the onset of entropy, whereas in my analysis a terminology of grey ages denotes political economies that are indifferent or hostile to higher intellectual effort.

No Lodge need accept that a stream of candidates is the core experience or that a liberating consciousness can be achieved exclusively by repetition or osmosis, as that is fundamentalist. Whether stock Masonic lectures or expositions of degree ceremonies are a fall back position is hair splitting as few go to the metatheory. There is a tension within Masonry as to the purpose, whether as a school of elementary morality or as a social-commercial network or centred on the study of meta-theory. A latent interaction of Alpha creative and Beta judgmental-rational mind-sets can raise the connectivity of the network's consciousness.

Having synthesised the threads, it is now time to remix their essential features by a case study leading into findings on the education of the Masonic mind and its statistical mechanics, then concluding this research paper with a statement of the emergent Theorem.

## **Case Study**

## Walter Leslie Wilmshurst (1867-1939)

Questions are posed, by the Lodge of Living Stones' *Life and Work of W.L. Wilmshurst* (1954), Antony Bakers' *W.L. Wilmshurst: His World of Fallen but Living Stones* (2006), and then WLWs *Tracing Board of the Centre* discovered by Robert Lomas in archived material at the University of Bradford. Is there more to WLW's network than previously evident? To what extent did established or yet to be defined threads in the sciences and social analysis print through, to such a well read and studious man, from the likes of George Boole (1818-1864), Bernhard Riemann (1826-1866), Georg Cantor (1845-1918), Bertrand Russell (1872-1970), John Keynes (1883-1946), Joseph Schumpeter (1883-1950), and others laying the ground work for advances in the future, such as the work of Edward Lorenz (1917-2008)? In any event, why not promote WLW's model from a static historical artefact, to a foundation stone on which to elaborate the knowledge of the statistical mechanics of the education of the Masonic mind?

Necessarily, mathematics and the sciences were in transition while WLW was busy with his Masonic writing, solicitor's practice, and the organisation, transcription and lecturing on choral music, like his famous radio talk on George Handel's (1685-1759) *Messiah*. Significant musical compositions have nonlinear intergenerational content, seen in their fractal like spiralling of intervals, sets and the decay of notes. Again, WLW in his Masonic essays refers to gnosis, the acquisition of a special kind of knowledge, noting that WLW did not have the discovery of ancient scrolls with their gnostic content on which to draw. Similarly, my area of research has barely begun to formulate the deeper questions for analysis.

#### Network

WLW's dedication to the mystical is underlined by his support of Mary Ann Atwood's (1817-1910) A Suggestive Inquiry into the Hermetic Mystery. First issued in 1850 the 1918 edition has an extensive contribution by WLW. The first edition was a limited circulation being recalled by her father Thomas South, a researcher in the field, in order to protect the hermetic secrets, although the language is a sufficiently complex cipher to only inform the adept.

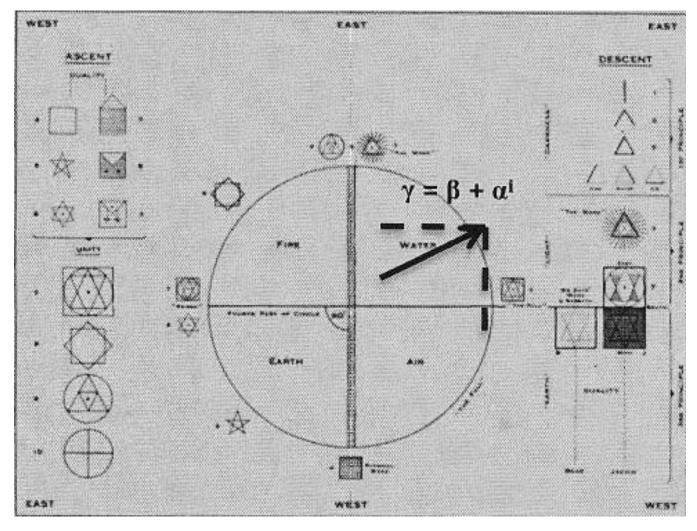
Hermes, the god of science and alchemy, may affront modern Freemasonry but has a valid historical context as with the Nag Hammadi texts with Gnostic content that is not monolithic. Masonic scholars are not necessarily Freemasons and the inverse applies in Bertrand Russell's paradox of set theory. Similarly, the certainty seeking Age of Enlightenment (circa 1650-1800 CE), brought a rationalist reading of society, up to a point, on the shoulders of the scholastic medieval academies gathering the knowledge of the 10

Mediterranean basin, Asia Minor, India and China of antiquity. Different currents sought variously to sustain or separate the spiritual, mystical, occult or the theosophy from science.

Could WLW's tracing board signify an awakening to a higher symbolism in mathematics such as George Boole's (1815-1864) algebra, noting WLW's concern about the naive and common attribution of Masonic symbolism as an end product? Boole as a Unitarian possibly holding some Gnostic beliefs might have resonated with WLW. An indirect intermediary could have been Boole's wife Mary Everest (1832-1916) through the Occult Society, which reviewed a book by Mary in 1910, at which time WLW was 43 and Mary 78 years old.

Mary Boole born to a clergyman, and niece of George Everest (1790-1866) the great trigonometric surveyor, had an interest in spirituality, also pioneering the psychology of mathematical education (a nice fit with George Boole who supported his research into algebraic logic by teaching) at a time when women could not obtain a university degree. Mary Boole was an exponent of the geometry of string art and the square root wheel, which recalls Islamic architecture. That anticipated the fractal self-replicating patterns systemised by Benoit Mandelbrot in *Fractals: Form, Chance and Dimensions* (1977), who had a long association with IBM. (Probability theory has a lot to do with computing due to the tunnelling properties of electrons through silicon screens.) The mathematical groundwork was prepared by the set theory and topology variously by Georg Cantor (1845-1918), Felix Hausdorff (1868-1942), Pierre Fatou (1878-1929), Waclaw Sierpinski (1882-1969), Gaston Julia (1893-1978), and applied by the geographer-statistician Jaromir Korcak (1895-1989).

WLW contributed many articles to the *Occult Review* between 1905 and 1914. Mary's work *The Forging of Passions into Power* was reviewed in August 1910 although whether she was a member of the society I do not know. Arthur Waite's (1857-1942) *History of Magic* was reviewed by WLW for the May 1913 edition. Karl von Eckartshausen (1752-1803), a favourite of WLW, previously wrote on *Magic: The Principles of Higher Knowledge*, 1788, then *The Cloud Upon the Sanctuary*, c1896. Eckartshausen's dissertation was brilliantly perceptive of magic as knowledge in plain view not seen because of the semiotics of society. All said the Masonic content of mystical writers produced an unresolved relationship with mathematics.



## Tracing board of the centre

Evident are the classic Greek elements and their combinational symbolism mapped in the algebra of George Boole. This recalls the mystical lineage of the yet to be rediscovered Hildegard von Bingen (1098-1179 CE), with her reservation about militarised crusaders. Hildegard was prescient of the 1204 CE sacking of Constantinople and then 54 years later Baghdad where al-Khwarizmi (c780-850 CE) formulated the algorithm. Now the tracing board appears as a play on entropy (descent), re-birthing (ascent) and the transitory equilibrium (unity), in a combinational Euclidian triangular duality. That points toward the Sierpinski carpet and the conditional possibilities in the spiral moment whose limit is  $\pi$ .

A sweeping Jean-Robert Argand (1768-1822) vector on a plane embeds complex numbers of alpha and beta brain states, as a building block for Benoit Mandelbrot self-replicating fractals connecting the structure. Cardinal (compass) points wrap around, geodesic like, in the set theory of Georg Cantor as nominal opposites prove the existence of the other. At their limits is a Masonic re-birthing in the dynamics of nonlinear system entropy (as a sand castle wants to deconstruct itself) in the chaotic reality of the mathematics of Henri Poincare then Edward Lorenz. Combinational Euclidian symbolism might reference the dualities of the truth tables and universal logic gates of George Boole. Perhaps there is a premonition of Kurt Godel's undecidable propositions (1931) or Bertrand Russell's paradox of set theory (1902), where only the unlikely probability of a closed set can contain all possible outcomes, and thus Karl Popper's (1902-1994) argument against the axiomatic.

WLW in his collection of essays, *The Meaning of Masonry* (1922), offers an allegorical explanation of the centre and the Christian axiom that the kingdom of heaven is within you. The tracing board appears to be a testing of the axiom of the enclosing circle, or a closed set. Had WLW absorbed the summation of contemporary mathematician Cassius Keyser or the work of George Boole, or the conditionality of Charles Peirce and Bertrand Russell on set theory? *The Builder Magazine* (1915-1930) of the National Masonic Research Society (USA) reviewed Keyser's writings on any number of occasions. Freemasons of the time might have also been aware of Bertrand Russell's *Introduction to Mathematical Philosophy* (1919)?

Trigonometry of the centre may be a simile for the ascending stairs of history's social capital. Perhaps WLW had the general concept of the rotation of the topology by Leonhard Euler's complex-imaginary numbers. Is there an emerging integration with the simplex fractals of Islamic art or the string art of the Boolean algebra of the mosaic pavement? Their invariance (like water in containers of differing shapes) agrees with the idea of an enduring alien belonging. WLW had a notion of the enveloping geodesic sphere as sides of the tracing board hyper-plane wrap around top centre to bottom centre and diagonally, but did he know of the formal simplex outer and complex inner structure of combinational topology?

Expression of Hausdorff fractal dimensions appears consistent with WLW's thinking. A theoretical limit of 3 compares with a self imposed structural limit of 2 for society and its Masonic sub-culture, functioning below a threshold implied by the Masonic pillars. General society's historical achievement is an H-D of 1.5 and Freemasonry, with its axiomatic assumptions, will plateau within the 1.5 to 2.0 band thus well short of their expectations.

Extrapolating, hourglass triangular sets with their dot point eccentricity describe a Felix Hausdorff structure. There the structure of a self-replicating fractal of the Waclaw Sierpinski kind is an exterior materialistic plane leading into the complex inner space. Neighbourhoods separated by the layers are disjunctional as their coordinates define the dot point strange attractor while potentially being a member of the set in the next neighbourhood. Such relationships are likened to Leonhard Euler's theorem (solution to the seven bridges of Konigsberg riddle) relating vertices, edges and faces of adjoining 3-D solid forms. Interchange between phases of societies and also between societies is defined by the quadratic discriminant of conic sections characterising the network's political-economy.

Boolean gateways between the gross polar states of *normalising society* and the transit to *metatheory content* signals the release from structures captured by social engineering. Although there is a view that socio-economic systems can be predictively modelled and the chaos effect is in the noise, Les Oxley in *Economics on the Edge of Chaos* (2004) on stock market performance contends that standard economic modelling may be a case of force fitting data.

WLW's symbolism might therefore be viewed in the context of the disseminating lags involved as theoretical progress is made in the broad sweep of mathematics and science. Georg Cantor for example was unaware of the full significance of his own contribution in set theory, which was a pattern for George Boole. Charles Peirce foreshadowed electrical switching of Boolean truth tables and in the string art of Mary Boole

is an insight of the self-replicating natural fractal also recalling non-European art music and Islamic architecture.

### **Masonic Education**

Morris Kline promoted mathematical education within a framework inclusive of the philosophy, cultural settings, the dialectical history of mathematical thought (the scholastic method), and the critical appreciation of content. Kline was critical of hard-wired, rote or incremental linear learning. Kline's focus on metaeducation within a self-observing philosophy adds to the spiral learning curve, the constructivist approach to learning and cognitive mapping. That is congruent with the scholastic approach to economics originating in classical Greece, recovered in the European Middle Ages and reformulated by Joseph Schumpeter in a magnum opus *The History of Economic Analysis* (1954). A fusion of econometrics, political-economy and ethics, this contrasts with the mechanical mercantile core of Adam Smith's *An Inquiry into the Nature and Causes of the Wealth of Nations* (1776).

Coincidental with the spiral curriculum of Jerome Bruner, my modelling frames a three-tiered algebraic process of discovery by the Masonic sojourner to master complex ideas:

- A simplex or closed system mechanically enacting rote learning; then
- An evolving equilibrium seeking phase that is iconic or perceptive; then
- A complex open form of undirected constructivist-discovery that is symbolic.

The complex form reiterates and integrates the cognitive levels within the empirical, symbolic and theoretical. Constructivist approaches to learning pioneered by John Dewey (1859-1952), Jean Piaget (1896-1980) and Lev Vygotsky (1896-1934) place interactive learning mediated by semiotics, within the social and cultural setting and the internalisation of ideas. Carl Jung's individuation is one possible outcome where the balanced individual emerges and appropriate knowledge is systematically internalised. In this benign semiotic process the student takes ownership of the intellectual tools or materials. This may be compared with the hard wiring of the allegorical tools of the speculative Mason. Yet critics of the discovery approach flag it's open to exploitation by either student or instructor. That said, leaders and teachers cannot command insightful learning including critical thinking, rather their role is to provide guidance, method, context and other supporting infrastructure.

Paul Kirschner, John Sweller and Richard Clark explain *Why minimal guidance during instruction does not work* (2006) for novice-intermediate students, pointing to immature working memory that is limited in apacity to short-term operations. Cognitive interaction is fragile while the prior long-term memory is limited. Bruner's theory however can be evolved for the cognitive architecture pioneered in 1948 by Edward Tolman (1886-1959, brother of mathematician Richard). Humans learn to take in facts later to be used holistically for problem solving that goes beyond a hard-wired response to an external stimulus. Ambiguities in the chronology have been used to support biases toward either the rote or discovery methods of teaching. Edward Tolman in the journal *Psychological Review* (1948, page 208) opined: *As Freud would have put it, to be able to learn to live according to the Reality Principle rather than according to the too narrow and too immediate Pleasure Principle*.

In practice, cultural transmission, whether of the innate or explicit kind, influenced by the provisional cultural environment of Karl Popper, has the capability to blend directed learning of the general concepts of knowledge domains at the novice-intermediate level, with a graduated deepening of domain and multi-disciplinary knowledge. As knowledge deepens, the emphasis goes to the analytical protocols while the student creates or restores meta-theory. Blending can be thought of as John Venn (1834-1923) diagrams that integrate directed and undirected learning systems and build cognitive connectivity.

Ritual's place in learning is not widely understood. Catherine Bell (1953-2008: Rituals, Dimensions and Perspectives, 1997) characterised ritual as a symbolic language encapsulating contradictions to orient ideals as a precursor for their cognitive adoption. Alexander Piatigorsky (1929-2009) in Who's Afraid of Freemasons (pages 289-90) posited that the Masonic degree is: not an initiation into a kind of Higher Knowledge, but into the knowledge of the ritual. My modelling suggests that Freemasonry revisit its spiral curriculum to distinguish between:

- Hubs developing the replicating ritual within a simplex macrostructure
- Hubs of research and critical thinking to support a complex microstructure

• Nodal Lodges that are academies integrating those symbolic structures

Their layering may be visualised by contrasting the formal geometry of the architecture of Greece of antiquity with the fractal edging of gothic architecture, that are attractors for non-linear formations, as in the gothic revival *Old St Paul's* Wellington New Zealand or the Mezquita Cathedral of Cordoba (Spain) fusion of the Islamic and Gothic styles. Such an infrastructure is in the tradition of the libraries and archives of Alexandria, Constantinople, Baghdad and today's physical and virtual stacks, supporting the rigorous multi-disciplinary search for a collective consciousness and its application by the practice of Freemasonry.

Leading Australian academics speak of science and mathematics as a foundation for a society equipped for critical thinking. The Australian Academy of Science, reporting on *The Status and Quality of Year 11 and 12 Science in Australian Schools* (2011), found a propensity to load students with uninspired rote information. Neuroscientist Professor Edward Byrne of Monash University (*Sink or Swim: Imperatives for Australian Universities in the Next Decade*, 2011) argues that tablet computers supersede memorising large tracts of information while the neural connectivity and interactive shaping requires a different approach to education.

### **Statistical Mechanics**

Now it is possible to bring the threads of the statistical mechanical analysis together. Patterns as with thermodynamic systems, depend upon whether statistical ensembles are: micro canonical (a closed system with limited degrees of freedom), or a canonical community (a system in equilibrium), or grand-canonical (an open system). Society's probable fractal achievements are located on the learning curve as an outgrowth of a Sierpinski fractal carpet.

Statistical mechanics, originally formulated by Ludwig Boltzmann in 1896, explains the aggregate of the probabilistic particles of matter in non-linear thermodynamic systems. As with many advances in physics, the theory is applied to social analysis, but that is not a straightforward exercise, as illustrated by the universes of Boltzmann and Josiah Gibbs. They variously proposed a homogenous distribution of pockets of entropy, or an expanding separation of nodes by low-density space depending upon time orientation. In my model the cultural structures are of the Archimedean power law (fractal) kind compared with the Fibonacci logarithmic inverse exponential of cognitive software. The latter is released from the normal curve's limited degrees of freedom into the multiple uncertainties and realities of the non-linear world, intensifying and shortening our mental connectivity.

	ALGEBRA OF THE MASONIC TOPOLOGY		
	HAUSDORFF DIMENSION	SPIRAL CURRICULUM	LODGING THRESHOLDS
	FRACTAL	CLOSED SET	A POSTERIOR &
	1.5	SIMPLEX : ROTE	NORMALISING
	FRACTAL	EVOLVING	PRO TEMPORE
	2.0	=	LJMIT
		ICONIC	@ RITUAL
		OPEN SET	A PRIORI
	FRACTAL	=	LIMIT IT
	2.8	COMPLEX:	MULTIPLE
		DISCOVERY	REALITIES

Respectful differences between contemporaries John Keynes and Joseph Schumpeter go to the probabilistic asymmetry of expectations, whether equilibrium as an end-state or the perpetual transition. Their perspectives go into the velocity of the circulation of economic factors. Schumpeter saw the statistical

evidence that the aggregate actions of individuals and corporations cannot be predicted with certainty in the trade cycle. Keynes held onto the notion of the demand driven multiplier that led him to propound a closer management of the economy to stabilise the trade cycle. Keynes' altruistic goal needed Schumpeter's theory.

Schumpeter's entrepreneur could be the supply side innovator (creative destruction), or a rent seeking free rider on the system, including interest groups and associations with their hierarchies within, some monopolistic or with barriers to entry. Schumpeter was not a laissez faire economist (like Adam Smith). Nor was he statistically atheist like Friedrich Hayek (1899-1992: *The Road to Serfdom*, 1944). They probably subscribed to *The Recollections* (part I, chapters 1 and 2) of Alexis de Tocqueville (1805-1859) on the self-interested and unintended consequences of collectivist actions with their imperfect knowledge. Bertrand Russell (*The Problems of Philosophy*, chapter 15) had a concern about the influence of practical men of mercantile character. Emergent is a consensus that the axiomatic *a posterior* diminishes society's capacity for the disinterested algebra in *a priori* complexities. Inevitably these positions interact, explained by the probability inference of Thomas Bayes (1702-1761). Cycling of preliminary *a priori* perceptions are updated by empirical evidence, and thus the symbolism and communication of invariant patterns of culture create new social capital.

#### **Theorem**

Rendering the symbolic topology of society are the collective choices by:

• Those lodging in an axiomatic and hard-wired Euclidian-Newtonian past;

&

• Interest groups and their Faustian fellow-travellers, lodging in the present, to appropriate the symbolism of the ritualised outer-structure;

&

• The disinterested lodging deep in the algebraic symbolism of the system connectivity and software dynamics.

Historically, homologating society tends toward ceding freedom to an *orderly harmonious arrangement*. That marginalises the potential of an *orderly understanding* of the dynamic and probabilistic *harmonics* of the collective mind. Such meta-knowledge and a liberated consciousness are found in the connectivity and algebraic symbolism of the Masonic Mind.

Moreover, the post-structuralist idea of semiotician Roland Barthes (1915-1980: *The Death of the Author*, 1967), with the author surrendering his work, is taken to a new level by the digital net casting of multimedia with content creatively merging consumer, producer and audience. The multi-media interface rewires the brain connectivity and learning methods so introducing new levels of flexibility and comprehension of dynamic systems. The Masonic lodge is a form of multi-media of an earlier analogue and hard-wired form, that is a fractal like macro-building block for the complex microstructure of the Masonic sojourner.

So the critical point is the probabilistic importance of the relationship of the surface macrostructure, to the network's inner dynamic microstructure, where the satisfaction of the normalised, is displaced by disinterested expectations.

The arrow of time is asymmetrical. Only the past event horizon is visible, and only the future is malleable. In the bounded universe of collective minds with the infinity of intervals, the journey is never completed and there are no ultimate answers, just better questions. Readers are encouraged to explore the reference material to gain an appreciation of the knowledge domains, to form an opinion of the way the material is interpreted and situated by the discussion, to improve or develop a better theorem of the dimensions of the Masonic mind.

## I conclude with these questions to the Masonic community:

- Is the symbolism of Masonic Lodging sufficiently algebraic?
- Do we adequately use or acquire skill sets to develop Masonic theory?
- What does this mean for the content and delivery of Masonic education?

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